Exercise 20

(a) If \$1000 is borrowed at 8% interest, find the amounts due at the end of 3 years if the interest is compounded (i) annually, (ii) quarterly, (iii) monthly, (iv) weekly, (v) daily, (vi) hourly, and (vii) continuously.

(b) Suppose \$1000 is borrowed and the interest is compounded continuously. If $A(t)$ is the amount due after t years, where $0 \leq t \leq 3$, graph $A(t)$ for each of the interest rates 6%, 8%, and 10% on a common screen.

Solution

Part (a)

The value of an investment that has compounding interest (n times per year with an interest rate r) is

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}.$$

The amount owed back at the end of 3 years if the interest is compounded annually is

$$A(3) = 1000 \left(1 + \frac{0.08}{1}\right)^{(1)3} \approx \$1259.71.$$

The amount owed back at the end of 3 years if the interest is compounded quarterly is

$$A(3) = 1000 \left(1 + \frac{0.08}{4}\right)^{(4)3} \approx \$1268.24.$$

The amount owed back at the end of 3 years if the interest is compounded monthly is

$$A(3) = 1000 \left(1 + \frac{0.08}{12}\right)^{(12)3} \approx \$1270.24.$$

The amount owed back at the end of 3 years if the interest is compounded weekly is

$$A(3) = 1000 \left(1 + \frac{0.08}{\frac{365}{7}}\right)^{\left(\frac{365}{7}\right)3} \approx \$1271.02.$$

The amount owed back at the end of 3 years if the interest is compounded daily is

$$A(3) = 1000 \left(1 + \frac{0.08}{365}\right)^{(365)3} \approx \$1271.22.$$

The amount owed back at the end of 3 years if the interest is compounded hourly is

$$A(3) = 1000 \left(1 + \frac{0.08}{24(365)}\right)^{[24(365)]3} \approx \$1271.25.$$

The amount owed back at the end of 3 years if the interest is compounded continuously is

$$A(3) = \lim_{n \to \infty} 1000 \left(1 + \frac{0.08}{n}\right)^{(n)3} = 1000e^{0.08(3)} \approx \$1271.25.$$

Part (b)

If the interest is compounded continuously, the amount owed back after t years is

$$A(t) = A_0 e^{rt},$$

where A_0 is the amount borrowed and r is the interest rate.